

Is the Higgs a sign of extra dimensions?

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Abstract

We introduce a 4-dimensional cutoff in the scenario of gauge-Higgs unification to control the ultraviolet behavior. A one-loop effective potential for a Higgs field and the Higgs mass are obtained with the cutoff. We find an *interrelation* between the 4-dimensional cutoff and the scale of extra dimensions, which is concretized through the Higgs mass. Combining this interrelation and the recently discovered Higgs boson at LHC, we obtain an interesting constraint for the 4-dimensional cutoff and the extra dimensional scale.

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1 Introduction

A higher dimensional gauge theory is one of the attractive candidates for physics beyond the standard model. Gauge-Higgs unification [1] is one of such the gauge theories, where gauge and Higgs fields are unified into a higher dimensional gauge field. Component gauge fields for compactified extra directions behave like the Higgs fields at low energy.

In the scenario of the gauge-Higgs unification, the gauge symmetry is broken through quantum corrections [2], and the Higgs mass, which is zero at the tree-level due to the higher dimensional gauge invariance, arises at quantum level. It has been said that the effective potential for the Higgs field and the Higgs mass do not suffer from ultraviolet divergences. Thanks to this property, the gauge-Higgs unification may solve the gauge hierarchy problem without relying on supersymmetry [3]. The gauge-Higgs unification has been an attractive alternative for the Higgs mechanism. Many attempts to seek phenomenologically viable models with the gauge-Higgs unification have been done in the past [4, 5, 6]. In addition to it, various aspects of the gauge-Higgs unification such as the finite temperature phase transition *et .al.* have also been studied [7, 8, 9, 10].

In the gauge-Higgs unification, one needs to evaluate the effective potential for the Higgs field in order to discuss the gauge symmetry breaking patterns and to calculate the Higgs mass which is obtained by the second derivative of the potential at the vacuum. In the past, one employed the dimensional regularization for the momentum integration in evaluating the effective potential at the one-loop level. The divergent terms that depend on the order parameter (the Higgs field) do not appear in the effective potential and the Higgs mass. But the dimensional regularization essentially can not account for power divergences.

As stated above, the Higgs mass arises through quantum corrections in extra dimensions, say, Kaluza-Klein modes in the gauge-Higgs unification. It is, however, difficult to obtain the definite quantum effect of the higher dimensional gauge theory because of the nonrenormalizability. The detailed structure of the effective potential for the Higgs field is unknown as long as one can not solve the dynamics in higher dimensions. At the moment, it remains unclear how much one should take the quantum correction in the extra dimension into account in order to determine the low-energy physics.

There appears an infinite number of the Kaluza-Klein mode in 4 dimensions at the tree level after the compactification. The ultraviolet contribution in the sense of 4-dimensional quantum field theory is controlled by introducing the 4-dimensional cutoff, so that the quantum correction from the Kaluza-Klein modes to the effective potential and the Higgs mass are well-defined.

It is reported that the new scalar particle is discovered at LHC, which is consistent with the Higgs boson in the standard model [11, 12]. Taking account of the discovery, it is natural to expect that there lies the 4-dimensional cutoff consistent with the discovery.

In this paper, we find a parameter called as *interrelation*, which concretely tells us how much quantum correction in the extra dimension one should take into account. The interrelation also specifies whether the 4-dimensional cutoff is much more effective or the 5-dimensional physical effects dominate. The interrelation becomes manifest through the Higgs mass, and the consistent scale of the cutoff is obtained by taking account of the LHC result on the Higgs mass.

The aforementioned interrelation is obtained in evaluating the effective potential at the one-loop level by introducing a 4-dimensional cutoff, which respects the gauge invariance for the direction of the extra dimension. It follows that there is a remarkable combination between the periodicity of the Higgs field, that is, the gauge invariance for the direction of the extra dimension and an exponential suppression with respect to the cutoff.

This paper is organized as follows. In the next section, after brief setup, we present the expression for one-loop effective potential for the Higgs field and the Higgs mass with the 4-dimensional cutoff. The interrelation, which is a key notion, is also explained. In section 3, we study the interrelation through the Higgs mass in some models with the gauge-Higgs unification. We give a constraint on the 4-dimensional cutoff and the scale the extra dimension by taking account of the result on the Higgs mass at LHC. The final section is devoted to the conclusions. In Appendix, important formulae used on the text are derived.

2 Effective potential and Higgs mass with 4-dimensional cutoff

Let us consider a nonsupersymmetric $SU(3)$ gauge theory on $M^4 \times S^1/Z_2$, where M^4 is the 4-dimensional Minkowski space-time and S^1/Z_2 is an orbifold³. One must specify boundary conditions of fields for the S^1 direction and the two orbifold fixed points at $y = 0, \pi R$, where R is the radius of the S^1 . They are defined by

$$A_{\hat{\mu}}(x^\mu, y + 2\pi R) = U A_{\hat{\mu}}(x^\mu, y) U^\dagger, \quad (1)$$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x^\mu, y_i - y) = P_i \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x^\mu, y_i + y) P_i^\dagger \quad (i = 0, 1), \quad (2)$$

where $U = U^\dagger$, $P_i^\dagger = P_i = P_i^{-1}$ and $y_0 = 0, y_1 = \pi R$. The coordinate $x^\mu (\mu = 0, \dots, 3)$ denotes the 4-dimensional Minkowski space time and y is the coordinate of the extra dimension. Since the translation U together with the reflection P_1 is equivalent to the reflection P_0 , so that there is a relation $U = P_1 P_0$. We take $P_i (i = 0, 1)$ to be fundamental projections.

In the scenario of the gauge-Higgs unification, the zero modes for A_y play an important role and behave Higgs fields at low energy. If the Higgs field develops the vacuum

³Notations used in this paper are the same as those in [13].

expectation value, the $SU(2) \times U(1)$ gauge symmetry is broken to the electromagnetic $U(1)_{em}$. One must choose the boundary conditions $P_{0,1}$ in such a way that the zero mode for A_y belongs to the fundamental representation under the $SU(2)$ gauge group. We choose $P_0 = P_1 = \text{diag.}(-1, -1, 1)$. Then the $SU(3)$ gauge symmetry is broken explicitly down to $SU(2) \times U(1)$ by the orbifolding. The zero modes for the gauge field are read off by Eq. (2) for the boundary condition by $P_{0,1}$.

The zero modes for A_μ are given by

$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & A_\mu^1 - iA_\mu^2 & 0 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A_\mu^8 \end{pmatrix}, \quad (3)$$

by which the residual gauge symmetry is clearly $SU(2) \times U(1)$. On the other hand, the zero mode for A_y is found to be

$$A_y^{(0)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & A_y^4 - iA_y^5 \\ 0 & 0 & A_y^6 - iA_y^7 \\ A_y^4 + iA_y^5 & A_y^6 + iA_y^7 & 0 \end{pmatrix}. \quad (4)$$

We observe that

$$\Phi \equiv \sqrt{2\pi R} \frac{1}{\sqrt{2}} \begin{pmatrix} A_y^4 - iA_y^5 \\ A_y^6 - iA_y^7 \end{pmatrix} \quad (5)$$

belongs to the fundamental representation under the $SU(2)$. The adjoint representation of the $SU(3)$ is decomposed under the $SU(2)$ into

$$\mathbf{8} \rightarrow \mathbf{3} + \mathbf{2} + \mathbf{2}^* + \mathbf{1}. \quad (6)$$

We understand how the gauge and Higgs fields are embedded into the higher dimensional gauge field.

By utilizing the $SU(2) \times U(1)$ degrees of freedom, the vacuum expectation value for the Higgs field is parametrized by

$$\langle A_y^6 \rangle = \frac{a}{gR}, \quad (7)$$

where g is the 5-dimensional gauge coupling and a is a real parameter. The parameter a is related with the Wilson line phase,

$$W = \mathcal{P} \exp \left(ig \oint_{S^1} dy \langle A_y \rangle \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i \sin(\pi a) \\ 0 & i \sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2). \quad (8)$$

The original gauge invariance, namely concerning with the fifth direction, guarantees that the Wilson line phase is mod 2. The gauge symmetry breaking patterns of the $SU(2) \times U(1)$ are classified by the values of a ,

$$SU(2) \times U(1) \rightarrow \begin{cases} SU(2) \times U(1) & \text{for } a = 0, \\ U(1) \times U(1)' & \text{for } a = 1, \\ U(1)_{em} & \text{for otherwise.} \end{cases} \quad (9)$$

The value of a is determined as the global minimum of the effective potential for the Higgs field.

In the scenario of the gauge-Higgs unification, one needs not only the matter fields that satisfy the periodic boundary condition (PBC), but also the ones that satisfy the antiperiodic boundary condition (APBC). They are distinguished by the parameter η ($= 1$ for PBC, -1 for APBC) [13]. In addition to them, we also consider the matter fields belonging to the large representation under the $SU(3)$ gauge group such as the adjoint representation. These are necessary ingredients for the viable model with the gauge-Higgs unification.

Now let us discuss the effective potential for the Higgs field, that is, order parameter a . The effective potential is generated by quantum corrections in the 5 dimensions, that is, Kaluza-Klein modes. We evaluate it at the one-loop level and follow the standard prescription to calculate the effective potential⁴. Then one must evaluate the expression of the form,

$$F(Qa, \delta) = \frac{1}{2L} \sum_{n=-\infty}^{\infty} \int_{-\Lambda}^{\Lambda} \frac{d^4 p}{(2\pi)^4} \ln \left[p_E^2 + \left(\frac{n + Qa - \frac{\delta}{2}}{R} \right)^2 \right], \quad (10)$$

where $Q = 1, 1/2$ for the adjoint, fundamental representation under the $SU(2)$, respectively. The parameter δ takes 0 (1) for the field with the PBC (APBC). We have introduced the 4-dimensional ultraviolet cutoff Λ in the momentum integration because in general this is a divergent quantity. The effective potential is given by collecting all the contributions of the fields in theory,

$$V_{eff} = \sum_{i=fields} (-1)^F N_{deg}^i F^i(Qa, \delta). \quad (11)$$

The F stands for the fermion number of the internal loop, and N_{deg}^i is the number of on-shell degrees of freedom for the relevant matter field.

We first sum up all the Kaluza-Klein modes,

$$\sum_{n=-\infty}^{\infty} \frac{2p_E R^2}{(Rp_E)^2 + (n + Qa - \frac{\delta}{2})^2} = L \times \frac{\sinh(Lp_E)}{\cosh(Lp_E) - \cos(2\pi(Qa - \frac{\delta}{2}))}, \quad (12)$$

where we have defined $L \equiv 2\pi R$ and used the formula,

$$\sum_{n=-\infty}^{\infty} \frac{1}{x^2 + (n + a)^2} = \frac{\pi}{x} \frac{\sinh(2\pi x)}{\cosh(2\pi x) - \cos(2\pi a)}. \quad (13)$$

Let us note that summing up all the Kaluza-Klein modes is consistent with the gauge invariance for the direction of the extra dimension. By integrating it with respect to p_E ,

⁴We take the Feynman gauge for the gauge sector.

we immediately have

$$\sum_{n=-\infty}^{\infty} \ln \left[p_E^2 + \left(\frac{n + Qa - \frac{\delta}{2}}{R} \right)^2 \right] = \ln \left[\cosh(LP_E) - \cos \left(2\pi \left(Qa - \frac{\delta}{2} \right) \right) \right]. \quad (14)$$

It can be shown that the integration constant does not depend the order parameter a , so that we have set it to be zero.

We second perform the 4-dimensional momentum integration,

$$\begin{aligned} F(Qa, \delta, \tilde{\Lambda}) &= \frac{1}{2L^5} \frac{2\pi^2}{\Gamma(2)(2\pi)^4} \int_0^{\tilde{\Lambda}} d\tilde{p}_E \tilde{p}_E^3 \ln \left[\cosh \tilde{p}_E - \cos \left(2\pi \left(Qa - \frac{\delta}{2} \right) \right) \right] \quad (15) \\ &= \frac{1}{(4\pi)^2 L^5} \left[-6 \left(\text{Li}_5(e^{2\pi i(Qa - \frac{\delta}{2})}) + c.c. \right) + 6 \left(\text{Li}_5(e^{2\pi i(Qa - \frac{\delta}{2}) - \tilde{\Lambda}}) + c.c. \right) \right. \\ &\quad + 6\tilde{\Lambda} \left(\text{Li}_4(e^{2\pi i(Qa - \frac{\delta}{2}) - \tilde{\Lambda}}) + c.c. \right) + 3\tilde{\Lambda}^2 \left(\text{Li}_3(e^{2\pi i(Qa - \frac{\delta}{2}) - \tilde{\Lambda}}) + c.c. \right) \\ &\quad \left. + \tilde{\Lambda}^3 \left(\text{Li}_2(e^{2\pi i(Qa - \frac{\delta}{2}) - \tilde{\Lambda}}) + c.c. \right) \right], \quad (16) \end{aligned}$$

where the dimensionless integration variable have been defined by $\tilde{p}_E \equiv LP_E$ in Eq. (15), and we have used the Polylogarithm defined in Eq. (48) in Appendix. Here we have ignored constant terms that do not depend on the order parameter a . The derivation of Eq. (16) is given in Appendix.

In Eqs. (15) and (16) we have introduced a dimensionless parameter $\tilde{\Lambda}$ which relates the 4-dimensional cutoff Λ and the energy scale of the extra dimension L^{-1} as

$$\tilde{\Lambda} = L\Lambda = \frac{\Lambda}{1/L} \equiv \xi. \quad (17)$$

The parameter ξ in Eq. (17) plays an important role in the low-energy physics. Let us call it as *interrelation* between a 4-dimensional physics and the extra dimension. Here we notice that $\tilde{\Lambda}$ stands for not only the cutoff, but also the contribution of the Kaluza-Klein mode, depending on the scale of Λ . Namely, the latter point of view is crucial for the interrelation, which will be discussed in the section 3, so that we shall use the different notation ξ when we emphasize the interrelation such as the calculation of the Higgs mass.

Originally the 5-dimensional dynamics is out of control due to the nonrenormalizability. A cutoff must be introduced to define the theory, and it lies in certain energy scale though it is unknown where it should be. One does not know how much we should take account of the quantum correction from the Kaluza-Klein mode in order to determine the low-energy physics. At present, the discovery of the Higgs boson has been reported [11, 12] and we expect the consistent cutoff with LHC result must lie in certain energy scale. Then the interrelation tells us how much quantum correction from the Kaluza-Klein mode one should take into account in order for the cutoff to be consistent with the LHC result. At

the one-loop level, the effective potential is written in terms of the interrelation and, as we will see concretely later, the interrelation becomes manifest through the Higgs mass.

The first term in the right hand side of Eq. (16) is well-known and has been obtained in the past calculation [14]. One observes that all the terms except for the first term have a remarkable combination of ξ and the order parameter⁵,

$$e^{2\pi i Q a - \xi}. \quad (18)$$

The combination is resulted by respecting the gauge invariance for the direction of the extra dimension, that is, the periodicity of the order parameter a and introducing the 4-dimensional cutoff in the momentum integration (15). The potentially dangerous order parameter dependent divergence disappears as $\xi(= \tilde{\Lambda})$ goes to infinity thanks to the exponential damping. Let us note that the exponential behavior of the cutoff (18) never appears in the dimensional regularization.

The combination (18) is traced back to Eq. (12). By setting $\delta = 0$, it is rewritten as

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \frac{2p_E R^2}{(Rp_E)^2 + (n + Qa)^2} &= L \times \frac{\sinh(Lp_E)}{\cosh(Lp_E) - \cos(2\pi Qa)} \\ &= L \times \left(1 + \left\{ \frac{e^{2\pi i Q a - \tilde{p}_E}}{1 - e^{2\pi i Q a - \tilde{p}_E}} + c.c. \right\} \right). \end{aligned} \quad (19)$$

Then the relevant quantity is obtained by the integral of the form,

$$I(\tilde{\Lambda}) \equiv \int_0^{\tilde{\Lambda}} dy f(y) e^{i\tilde{a}-y}, \quad (20)$$

where the function $f(y)$ is an n -th polynomial, $f(y) = \sum_{k=1}^n a_k y^k$. The above integral is evaluated as

$$I(\tilde{\Lambda}) = F(0)e^{i\tilde{a}} - F(\tilde{\Lambda})e^{i\tilde{a}-\tilde{\Lambda}} = I(\infty) - F(\tilde{\Lambda})e^{i\tilde{a}-\tilde{\Lambda}}. \quad (21)$$

Here we have defined

$$F(y) \equiv \sum_{m=0}^n f^{(m)}(y). \quad (22)$$

The first term in Eq. (21) corresponds to the well-known finite term obtained in the past calculation. It is interesting to note that the ultraviolet limit of the function $I(\tilde{\Lambda})$ is evaluated at the infrared point of the integration variable $y = 0$ for another function $F(y)$. This is a notable feature in the scenario of the gauge-Higgs unification.

The effective potential is a special quantity in the gauge-Higgs unification because of the combination $e^{2\pi i Q a - \xi}$ at least at the one-loop level, which is never observed in the usual quantum field theory. Once we recognize this point, one immediately realizes

⁵The boundary condition δ of the field is not essential in this discussion, so that we have ignored it.

that quantity other than this type does not possess such the combination and hence the finiteness. As we will see below, the Higgs mass also has the same combination.

Now let us proceed to the Higgs mass, which is obtained by the second derivative of the effective potential at the vacuum denoted by $a = a_0$,

$$m_H^2 \equiv \left. \frac{\partial^2 V_{eff}}{\partial \langle A_y^6 \rangle^2} \right|_{vac} = (gR)^2 \left. \frac{\partial^2 V_{eff}}{\partial a^2} \right|_{a=a_0}. \quad (23)$$

The structure of the second derivative of the effective potential can be seen from Eq. (16) by

$$\begin{aligned} \frac{\partial^2 V_{eff}}{\partial a^2} &\propto \frac{\partial^2 F(Qa, \delta, \xi)}{\partial a^2} \\ &\propto -6 \left(\text{Li}_3(e^{2\pi i(Qa - \frac{\delta}{2})}) + c.c. \right) + 6 \left(\text{Li}_3(e^{2\pi i(Qa - \frac{\delta}{2}) - \xi}) + c.c. \right) \\ &+ 6\xi \left(\text{Li}_2(e^{2\pi i(Qa - \frac{\delta}{2}) - \xi}) + c.c. \right) + 3\xi^2 \left(\text{Li}_1(e^{2\pi i(Qa - \frac{\delta}{2}) - \xi}) + c.c. \right) \\ &+ \xi^3 \left(\text{Li}_0(e^{2\pi i(Qa - \frac{\delta}{2}) - \xi}) + c.c. \right). \end{aligned} \quad (24)$$

As stated before, we confirm that the Higgs mass also possesses the same combination, $e^{2\pi i Qa - \xi}$ as that of the effective potential.

If one takes the infinite limit of ξ , only the first finite term in Eq. (16) is survived to reproduce the well-known expression for the Higgs mass. In order to make discussions on the interrelation concretely, we need to consider models explicitly, which will be given in the next section.

3 Higgs as Interrelation between 4 and extra dimensions

Let us introduce a set of matter. We follow the studies of the gauge-Higgs unification made in the past [7, 13, 15], in which we have introduced the fermions and bosons satisfying the periodic boundary condition ($\eta = 1$) and antiperiodic boundary condition ($\eta = -1$), and whose representations under the $SU(3)$ gauge group are the adjoint and fundamental ones. We denote their flavor numbers by

$$(N_F^{adj(+)}, N_F^{fd(+)}, N_S^{adj(+)}, N_S^{fd(+)}), (N_F^{adj(-)}, N_F^{fd(-)}, N_S^{adj(-)}, N_S^{fd(-)}). \quad (25)$$

Here the $N_{F(S)}^{adj(fd)}$ stands for the number of the fermion (scalar) belonging to the adjoint (fundamental) representation under the $SU(3)$ gauge group. The \pm sign associated with $N_{F(S)}^{adj(fd)}$ is the periodicity of the matter field, $\eta = \pm 1$.

Recalling the equation (11), the effective potential with these types of matter fields

are given by

$$\begin{aligned}
V_{eff}^{total} = & \frac{1}{(4\pi)^2 L^5} \left[(-1)^0 3 J^{adj(+)} + (-1)^1 4 N_F^{adj(+)} J^{adj(+)} + (-1)^1 4 N_F^{fd(+)} J^{fd(+)} \right. \\
& + (-1)^0 4 N_S^{adj(+)} J^{adj(+)} + (-1)^0 4 N_S^{fd(+)} J^{fd(+)} + (-1)^1 4 N_F^{adj(-)} J^{adj(-)} \\
& \left. + (-1)^1 4 N_F^{fd(-)} J^{fd(-)} + (-1)^0 4 N_S^{adj(-)} J^{adj(-)} + (-1)^0 4 N_S^{fd(-)} J^{fd(-)} \right], \quad (26)
\end{aligned}$$

where the first term is the contribution from the gauge bosons, and we have defined

$$J^{adj(+)} \equiv F^\infty(2a, 0) + F^\xi(2a, 0, \xi) + 2(F^\infty(a, 0) + F^\xi(a, 0, \xi)), \quad (27)$$

$$J^{adj(-)} \equiv F^\infty(2a, 1) + F^\xi(2a, 1, \xi) + 2(F^\infty(a, 1) + F^\xi(a, 1, \xi)), \quad (28)$$

$$J^{fd(+)} \equiv F^\infty(a, 0) + F^\xi(a, 0, \xi), \quad (29)$$

$$J^{fd(-)} \equiv F^\infty(a, 1) + F^\xi(a, 1, \xi) \quad (30)$$

and

$$F^\infty(x, \delta) = -6 \left(\text{Li}_5(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2})}) + c.c. \right), \quad (31)$$

$$\begin{aligned}
F^\xi(x, \delta, \xi) = & 6 \left(\text{Li}_5(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) + 6\xi \left(\text{Li}_4(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) \\
& + 3\xi^2 \left(\text{Li}_3(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) + \xi^3 \left(\text{Li}_2(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right). \quad (32)
\end{aligned}$$

The shape of the effective potential is determined once we fix the number of flavor and ξ . In the limit of $\xi \rightarrow \infty$, the $F^\xi(x, \delta, \xi)$ vanishes, and the effective potential is given by the function $F^\infty(x, \delta)$ alone, which is consistent with the results obtained in the past calculation. The effective potential vanishes at $\xi = 0$, as seen from Eqs. (31) and (32).

Let us also give the second derivative of the effective potential which is necessary for the calculation of the Higgs mass by Eq. (23).

$$\begin{aligned}
\frac{\partial^2 V_{eff}^{total}}{\partial a^2} = & \frac{(2\pi)^2}{(4\pi)^2 L^5} (-1) \left[(-1)^0 3 J_H^{adj(+)} + (-1)^1 4 N_F^{adj(+)} J_H^{adj(+)} + (-1)^1 4 N_F^{fd(+)} J_H^{fd(+)} \right. \\
& + (-1)^0 4 N_S^{adj(+)} J_H^{adj(+)} + (-1)^0 4 N_S^{fd(+)} J_H^{fd(+)} + (-1)^1 4 N_F^{adj(-)} J_H^{adj(-)} \\
& \left. + (-1)^1 4 N_F^{fd(-)} J_H^{fd(-)} + (-1)^0 4 N_S^{adj(-)} J_H^{adj(-)} + (-1)^0 4 N_S^{fd(-)} J_H^{fd(-)} \right], \quad (33)
\end{aligned}$$

where we have defined

$$J_H^{adj(+)} \equiv F_H^\infty(2a, 0) + F_H^\xi(2a, 0, \xi) + \frac{1}{4} \times 2(F_H^\infty(a, 0) + F_H^\xi(a, 0, \xi)), \quad (34)$$

$$J_H^{adj(-)} \equiv F_H^\infty(2a, 1) + F_H^\xi(2a, 1, \xi) + \frac{1}{4} \times 2(F_H^\infty(a, 1) + F_H^\xi(a, 1, \xi)), \quad (35)$$

$$J_H^{fd(+)} \equiv \frac{1}{4} \left(F_H^\infty(a, 0) + F_H^\xi(a, 0, \xi) \right), \quad (36)$$

$$J_H^{fd(-)} \equiv \frac{1}{4} \left(F_H^\infty(a, 1) + F_H^\xi(a, 1, \xi) \right) \quad (37)$$

and

$$F_H^\infty(x, \delta) = -6 \left(\text{Li}_3(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2})}) + c.c. \right), \quad (38)$$

$$\begin{aligned} F_H^\xi(x, \delta, \xi) &= 6 \left(\text{Li}_3(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) + 6\xi \left(\text{Li}_2(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) \\ &+ 3\xi^2 \left(\text{Li}_1(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right) + \xi^3 \left(\text{Li}_3(e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}) + c.c. \right). \end{aligned} \quad (39)$$

The $F_H^\xi(x, \delta, \xi)$ vanishes for $\xi \rightarrow \infty$ to reproduce the old results for the Higgs mass, which is given by Eq. (38). At $\xi = 0$, the Higgs mass vanishes, as seen from Eqs. (38) and (39). The Higgs mass is given by ⁶

$$m_H^2 = (gR)^2 \frac{\partial^2 V_{eff}}{\partial a^2} \Big|_{a=a_0} = \frac{(2\pi gR)^2}{(4\pi)^2 L^5} H(Qa_0, \delta, \xi) = \frac{g_4^4}{(8\pi^2)^2} \left(\frac{v}{a_0} \right)^2 H(Qa_0, \delta, \xi), \quad (40)$$

where we have used the relation $v = a_0/g_4 R$ followed from the weak gauge boson mass $M_W = a_0/2R$, and we have defined $H(Qa, \delta, \xi)$ by the expression aside from the factor $(2\pi)^2/(4\pi)^2 L^5$ in Eq. (33). The 4-dimensional gauge coupling is defined by $g_4 \equiv g/\sqrt{L}$. The value of the Higgs mass is determined by putting the values of a_0, ξ and the number of flavor.

Now let us first study the case, where the matter content is given by

$$\text{model A : } \begin{cases} (N_F^{adj(+)}, N_F^{fd(+)}, N_S^{adj(+)}, N_S^{fd(+)}) = (2, 2, 0, 0), \\ (N_F^{adj(-)}, N_F^{fd(-)}, N_S^{adj(-)}, N_S^{fd(-)}) = (2, 2, 0, 3). \end{cases} \quad (41)$$

We first present the typical shape of the effective potential for $\tilde{\Lambda} \rightarrow \infty$ in Fig. 1.

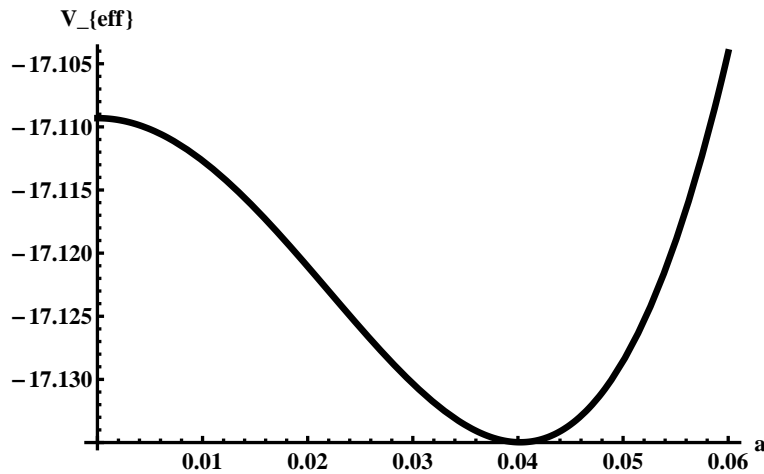


Figure 1: The shape of the effective potential in the limit of $\tilde{\Lambda} \rightarrow \infty$ for the model A. The global minimum is located at $a_0 = 0.0402199$.

⁶The models of the gauge-Higgs unification in this paper do not predict the correct the Weinberg angle, and we implicitly assume that we have made the prescription done, for example, in [16], so that the 4-dimensional gauge coupling becomes free parameter and that its size is of order of one.

The global minimum is located at $a_0 = 0.0402199$ and the $SU(2) \times U(1)$ gauge symmetry breaks down to $U(1)_{em}$. By using the vacuum expectation value a_0 , the Higgs mass in the same limit is calculated as $m_H/g_4^2 = 130.222[\text{GeV}]$. It has been known that the matter content is crucial for obtaining the sufficiently heavy Higgs mass [15].

Now we turn on the cutoff $\tilde{\Lambda}(= \xi)$. The shape of the effective potential is changed according to the value of $\tilde{\Lambda}$, so that the position of the global minimum is also changed. We show the behavior of a_0 with respect to $\tilde{\Lambda}$ in Fig. 2. The gauge symmetry is correctly broken, that is, $a_0 \neq 0, 1$ for the range of $\tilde{\Lambda}$ we have studied ⁷. The magnitude of $\tilde{\Lambda}$ for $\tilde{\Lambda} \gtrsim 10$ almost saturates the values obtained in the limit of $\tilde{\Lambda} \rightarrow \infty$.

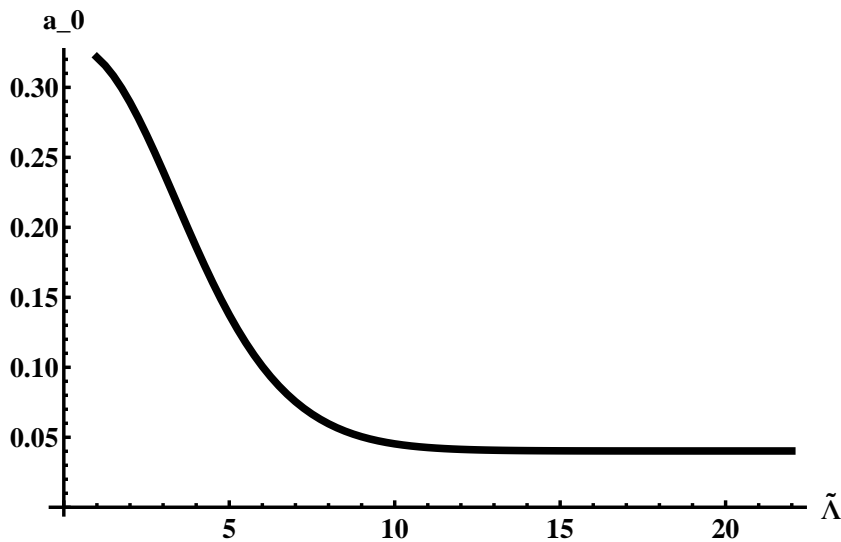


Figure 2: The behavior of the order parameter a with respect to $\tilde{\Lambda}$ for the model A.

Let us next depict the behavior of the Higgs mass with respect to the interrelation $\xi = \frac{\Lambda}{1/L}$ in Fig. 3.

We observe that the Higgs mass becomes larger as ξ is larger and for $\xi \gtrsim 10$ the Higgs mass is almost saturates the value obtained in the limit of $\xi \rightarrow \infty$. On the other hand, for $1 \lesssim \xi \lesssim 8$, the Higgs mass grows almost linearly with respect to $\tilde{\Lambda}$. If we take account of the recently reported Higgs mass, 126 [GeV] at LHC [11, 12], we obtain a bound on ξ . It is given by $\xi = \frac{\Lambda}{1/L} \gtrsim 10$, which implies that the 4-dimensional cutoff Λ must satisfy $\Lambda \gtrsim 10L^{-1}$. The value of the Higgs mass is smoothly connected to zero for $\xi \rightarrow 0$ as far as our numerical analyses are concerned.

We can also understand the behavior of the Higgs mass with respect to ξ by the first

⁷At $\tilde{\Lambda} = 0$, the effective potential vanishes, so that the position of the global minimum in the limit is unclear.

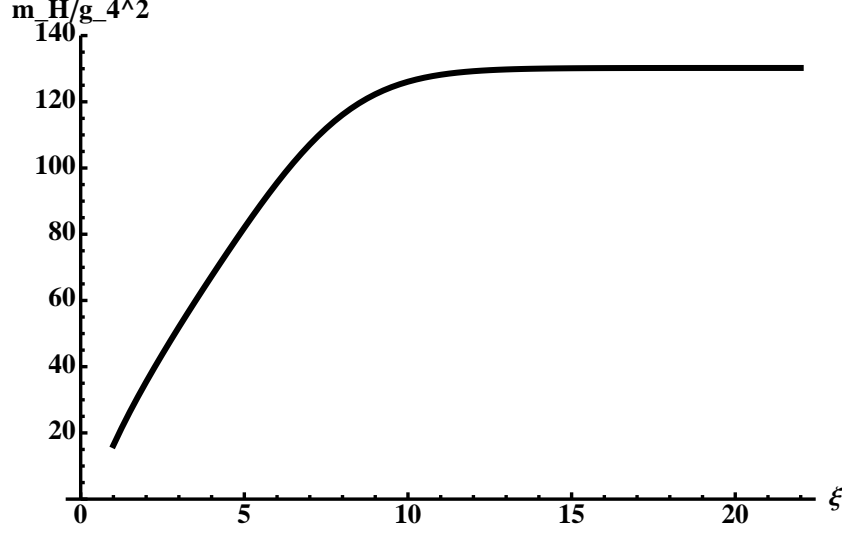


Figure 3: The behavior of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$ for the model A. The asymptotic value of the Higgs mass is about 130[GeV].

derivative of $F_H^\xi(x, \delta, \xi)$, which controls the Higgs mass essentially. It is given by

$$\begin{aligned} \frac{\partial F_H^\xi(x, \delta, \xi)}{\partial \xi} &= -\xi^3 \sum_{n=1}^{\infty} n \left(e^{2\pi i n (\frac{x}{2} - \frac{\delta}{2}) - n\xi} + c.c. \right) \\ &= -\xi^3 \left[\frac{e^{2\pi i (\frac{x}{2} - \frac{\delta}{2}) - \xi}}{\left(1 - e^{2\pi i (\frac{x}{2} - \frac{\delta}{2}) - \xi}\right)^2} + c.c. \right]. \end{aligned} \quad (42)$$

For large value of ξ , due to the exponential damping factor, the first derivative vanishes, so that the value of the Higgs mass becomes constant. This corresponds to the flat behavior in Fig. 3. When ξ becomes larger from zero, the ξ^3 starts to control the behavior of the Higgs mass. This gives the almost linear growth of the Higgs mass with respect to ξ in Fig. 3.

Let us discuss the interrelation $\xi = \frac{\Lambda}{1/L}$ which is manifest through the Higgs mass. If the 4-dimensional cutoff Λ is smaller than the the scale of the extra dimension, $\Lambda < L^{-1}$, the Kaluza-Klein modes can not be excited in the 4 dimensions. Since the Higgs mass is essentially generated by the quantum effect of the Kaluza-Klein mode, the Higgs mass is tiny enough for the region of the scale $\xi \lesssim 1$. As the cutoff Λ becomes larger, the Kaluza-Klein modes can start to excite and contribute to the Higgs mass, so that it gradually becomes heavier. This corresponds to the slop in the region of $1 \lesssim \xi \lesssim 8$. When the Λ becomes large further, $L^{-1} \leq \Lambda$, the Kaluza-Klein modes can be excited fully enough to yield the Higgs mass corresponding to the flat part. The behavior of the Higgs mass clearly shows the interrelation between the effect of the 4-dimensional cutoff and the physics in 5 dimensions, that is, the Kaluza-Klein mode.

As an illustration, let us also consider the two more cases, where the matter contents

are given by

$$\text{model B : } \begin{cases} (N_F^{adj(+)}, N_F^{fd(+)}, N_S^{adj(+)}, N_S^{fd(+)}) = (3, 2, 0, 0), \\ (N_F^{adj(-)}, N_F^{fd(-)}, N_S^{adj(-)}, N_S^{fd(-)}) = (4, 1, 1, 3). \end{cases} \quad (43)$$

$$\text{model C : } \begin{cases} (N_F^{adj(+)}, N_F^{fd(+)}, N_S^{adj(+)}, N_S^{fd(+)}) = (3, 4, 0, 0), \\ (N_F^{adj(-)}, N_F^{fd(-)}, N_S^{adj(-)}, N_S^{fd(-)}) = (5, 1, 2, 4). \end{cases} \quad (44)$$

In the limit of $\tilde{\Lambda} \rightarrow \infty$, the Higgs mass in the model B (C) is 186.694(168.096)[GeV], where the order parameter at the vacuum is given by $a_0 = 0.0285365(0.0436442)$.

We turn on the cutoff $\tilde{\Lambda}$ and depict the behavior of the order parameter a_0 in Fig. 4 for the models B and C. For the range of $\tilde{\Lambda}$, we have studied the gauge symmetry is broken correctly. In Fig. 5, we show the behaviors of the Higgs mass for the two models. For the model B (C), if we take account of the LHC result of the Higgs mass 126 [GeV], we obtain $\xi = \frac{\Lambda}{1/L} \gtrsim 5.7(6.26)$, which implies $\Lambda \gtrsim 5.7(6.26)L^{-1}$.

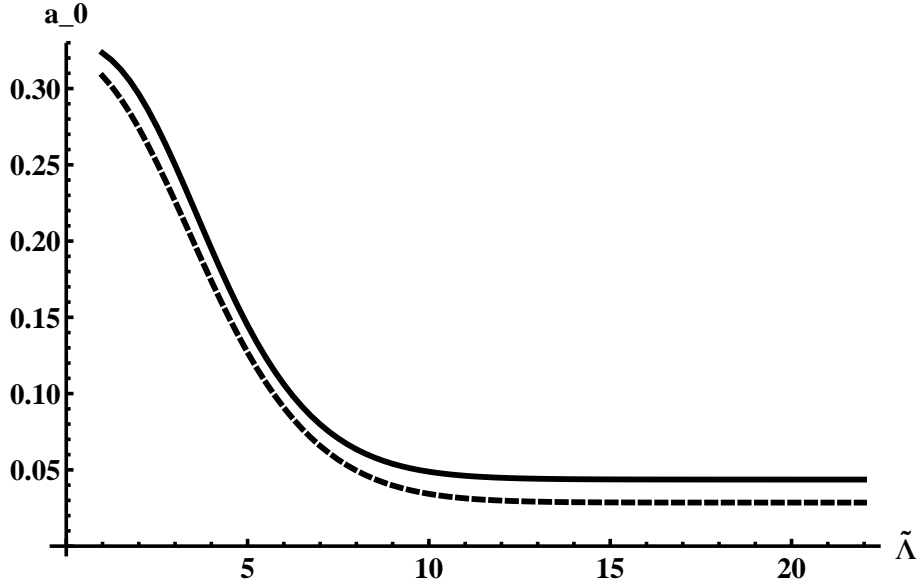


Figure 4: The behavior of the order parameter a with respect to $\tilde{\Lambda}$. The dotted (solid) line stands for the case of model C (B).

4 Conclusions

We have evaluated the one-loop effective potential and the Higgs mass in the scenario of the gauge-Higgs unification by introducing the 4-dimensional cutoff Λ in order to control the ultraviolet effect. It is clarified how much Kaluza-Klein mode appeared in 4 dimensions contributes to the effective potential and the Higgs mass thanks to the cutoff. The effective potential and the Higgs mass depend on both the order parameter a and $\xi \equiv \frac{\Lambda}{1/L}$ through

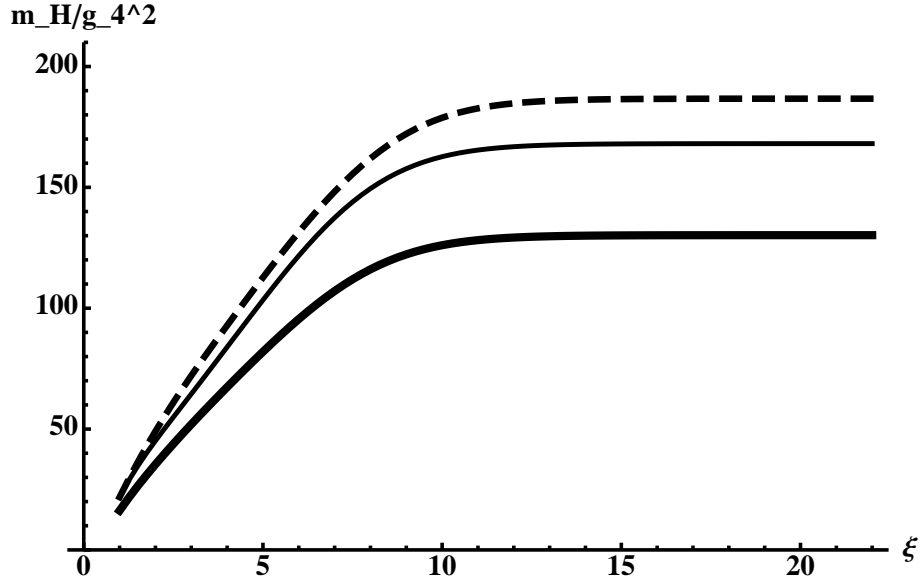


Figure 5: The behavior of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$. The thick (thin, dashed) line is the case for the model A (C, B). The asymptotic value of the Higgs mass for each model is about 130(168, 186)[GeV] for model A (C, B).

the remarkable combination $e^{2\pi i Q a - \xi}$. Due to the exponential damping, the well-known terms that obtained in the past calculations are reproduced in the limit of $\xi \rightarrow \infty$.

The parameter $\xi = \frac{\Lambda}{1/L}$ stands for the interrelation, which is, in particular, concretized through the Higgs mass. We have presented the three models in order to study the interrelation. We have obtained the behaviors of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$. The behavior shows the interrelation between the 4 and the extra dimensions. For the smaller cutoff Λ , the Kaluza-Klein excitations are suppressed in 4 dimensions, so that the Higgs mass, which essentially originates from the quantum effect of the Kaluza-Klein mode, is suppressed as well. As the cutoff Λ becomes larger, the excitations can be allowed to generate the Higgs mass gradually and for the certain large value of Λ , the Higgs mass approaches to the value obtained in the limit of $\Lambda \rightarrow \infty$, which means that the quantum correction in the extra dimension is fully incorporated. The interrelation is manifest through the Higgs mass, which shows that the 5-dimensional effect dominates for the large Λ , while the 4-dimensional cutoff becomes effective for the smaller Λ .

We have also obtained the bound on ξ by taking account of the LHC result. This, in turn, gives the bound on the ratio between the 4-dimensional cutoff Λ and the scale of the extra dimensions $1/L$.

The combination $e^{2\pi i Q a - \xi}$ is remarkable if we think of the usual logarithm and power behaviors with respect to the cutoff in the quantum field theory. The combination shows that the effective potential and Higgs mass are the special quantities in the gauge-Higgs unification. The origin of the combination may be the gauge invariance in the extra dimension. It is interesting to ask whether such the combination still holds beyond the

one-loop calculation [17] and to investigate the role of the combination further. It may shed new light on the gauge-Higgs unification from a point of view of quantum field theory. These will be reported in elsewhere [18].

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Appendix

Derivation of Eq. (16)

The momentum integration in Eq. (15) can be performed analytically. It is easy to show that the indefinite integration is carried out as

$$\begin{aligned}
& \int dy \, y^3 \ln(\cosh y - \cos \bar{a}) = \int dy \left(\frac{y^4}{4} \right)' \ln(\cosh y - \cos \bar{a}) \\
&= \frac{y^4}{4} \ln(\cosh y - \cos \bar{a}) - \int dy \, \frac{y^4}{4} \left(1 + \left\{ \frac{e^{i\bar{a}-y}}{1 - e^{i\bar{a}-y}} + c.c. \right\} \right) \\
&= \frac{y^4}{4} \ln[\cosh y - \cos \bar{a}] - \frac{y^5}{20} - \frac{y^4}{4} (\ln(1 - e^{i\bar{a}-y}) + c.c.) \\
&\quad + \int dy \, y^3 (\ln(1 - e^{i\bar{a}-y}) + c.c.). \tag{45}
\end{aligned}$$

It is straightforward to show that the first three terms in Eq. (45) become

$$\begin{aligned}
& \frac{y^4}{4} \ln[\cosh y - \cos \bar{a}] - \frac{y^5}{20} - \frac{y^4}{4} (\ln[1 - e^{i\bar{a}-y}] + c.c.) \\
&= \frac{-y^5}{20} + \frac{y^4}{4} \ln \left(\frac{\cosh y - \cos \bar{a}}{(1 - e^{-i\bar{a}-y})(1 - e^{-\bar{a}-y})} \right) \\
&= \frac{-y^5}{20} + \frac{y^4}{4} \ln \left(\frac{e^y}{2} \right) = \frac{y^5}{5} - \frac{\ln 2}{4} y^4. \tag{46}
\end{aligned}$$

In the second line of Eq. (45) we first expand the logarithm by ⁸

$$\ln(1 - x) = - \sum_{n=1}^{\infty} \frac{x^n}{n}, \tag{47}$$

and after the partial integration we make use of the Polylogarithm,

$$\text{Li}_s(z) \equiv \sum_{n=1}^{\infty} \frac{z^n}{n^s}. \tag{48}$$

⁸Note that the mode n in Eqs. (47) and (48) is different from the original Kaluza-Klein mode n . We point out that the mode summation (47) is the same as the one obtained by the Poisson's resummation formula.

We finally obtain that

$$\begin{aligned}
\int dy y^3 \ln [\cosh y - \cos \bar{a}] &= \frac{y^5}{5} - \frac{\ln 2}{4} y^4 \\
&+ y^3 (\text{Li}_2(e^{i\bar{a}-y}) + c.c.) + 3y^2 (\text{Li}_3(e^{i\bar{a}-y}) + c.c.) \\
&+ 6y (\text{Li}_4(e^{i\bar{a}-y}) + c.c.) + 6 (\text{Li}_5(e^{i\bar{a}-y}) + c.c.). \quad (49)
\end{aligned}$$

The first and second terms, which are independent on \bar{a} and something like the cosmological constant. Equipped with Eq. (49), the momentum integration (15) is evaluated as Eq. (16).

The momentum integration for the case of $M^{D-1} \times S^1/Z_2$ is also carried out in the same manner. It is given by

$$\begin{aligned}
&\int dy y^{D-2} \ln (\cosh y - \cos \bar{a}) = \int dy \left(\frac{y^{D-1}}{D-1} \right)' \ln (\cosh y - \cos \bar{a}) \\
&= \frac{y^{D-1}}{D-1} \ln (\cosh y - \cos \bar{a}) - \int dy \frac{y^{D-1}}{D-1} \left(1 + \left\{ \frac{e^{i\bar{a}-y}}{1 - e^{i\bar{a}-y}} + c.c. \right\} \right) \\
&= \frac{y^D}{D} - \frac{\ln 2}{D-1} y^{D-1} \\
&+ y^{D-2} (\text{Li}_2(e^{i\bar{a}-y}) + c.c.) + (D-2)y^{D-3} (\text{Li}_3(e^{i\bar{a}-y}) + c.c.) \\
&+ (D-2)(D-3)y^{D-4} (\text{Li}_4(e^{i\bar{a}-y}) + c.c.) \\
&+ (D-2)(D-3)(D-4)y^{D-5} (\text{Li}_5(e^{i\bar{a}-y}) + c.c.) + \dots \\
&+ (D-2)(D-3)\dots(D-(D-2))(D-(D-1)) (\text{Li}_D(e^{i\bar{a}-y}) + c.c.). \quad (50)
\end{aligned}$$

$D = 5$ is our case (49).

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